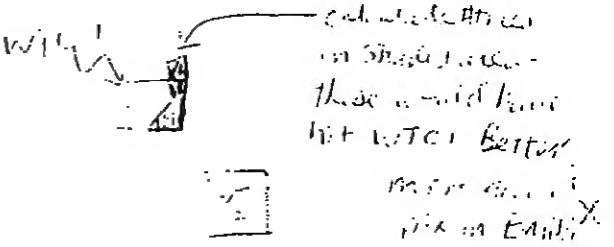
SIMPSON GUMPERTZ & HEGER INC.
ARLINGTON, MASSACHUSETTS SAN FRANCISCO, CALIFORNIA

SHEET NO. 1
COMM. NO. 22036.00 - WTC
DATE 8 Apr 2002
BY Brian D. Rose
CHECK BY

CLIENT Williams & Connelly
SUBJECT Swiss Re vs. World Trade Center Properties, et al
Calcs to determine number of hits stustained by remaining tower

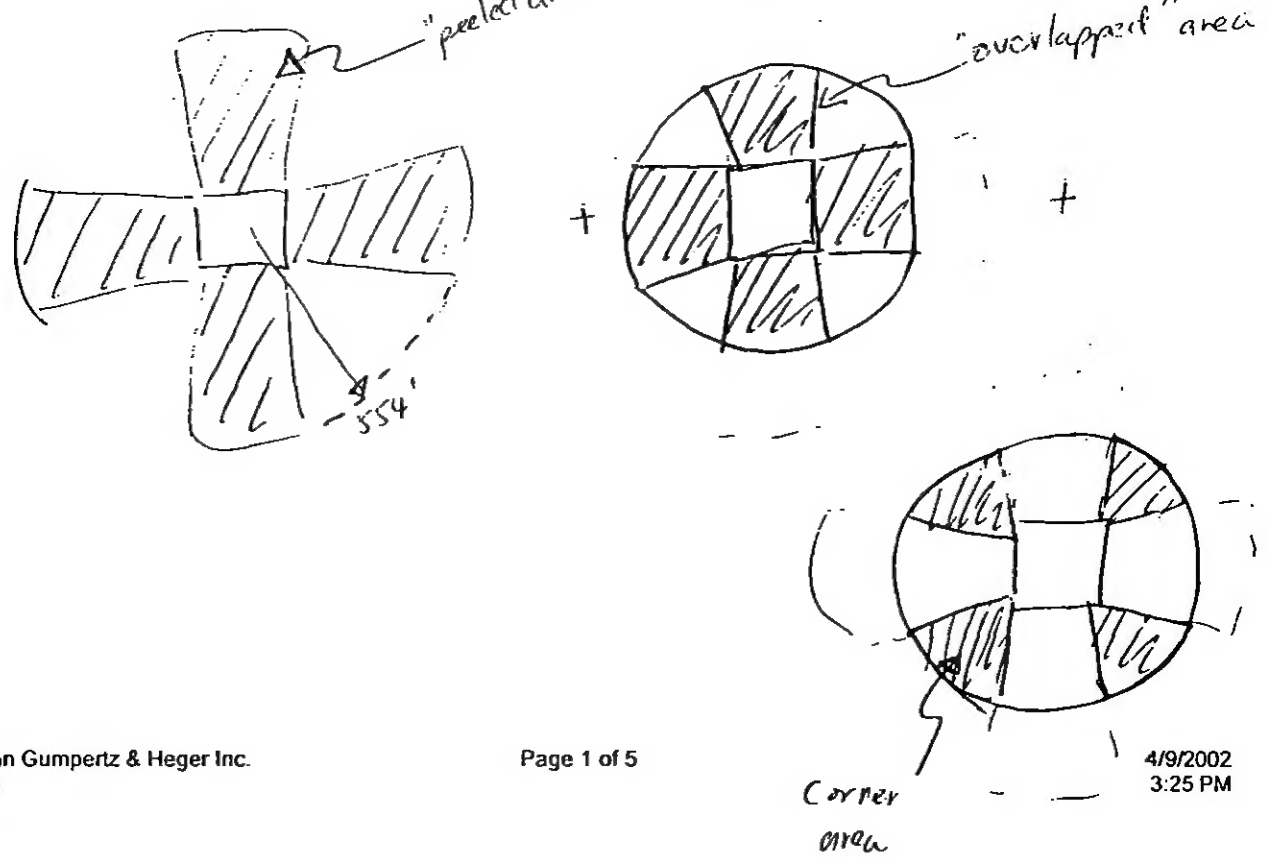


Inputs

Spread of trajectory	$\theta_{spread} := 15deg$
Length of side of tower	$L_{tower} := 208ft$
Radius	$r_{outer} := 554ft$
Radius	$r_{inner} := 400ft$
Number of trees per face	$N_{tree.per.face} := 700$
Tower total height	$h_{total} := 1361ft$
Height of skin that "peeled"	$h_{peel} := 460ft$
Percentage of debris not counted in "peeled" area that fell into the "overlapped" area	$Ratio_{overlap.to\ others} := 69.8\%$

critical case where tree density in "overlapped" area is same as "corner" area

Debris pattern



Calculate area in one petal

Solve for intersection of circle and line representing flare
Origin is at center of circle

Radius of interest	$r := r_{outer}$	$r = 554.00\,ft$
Initial guess for x	$x := r$	
Initial guess for y	$y := 0\,ft$	
Tolerance for solver	$TOL := 1 \cdot 10^{-6}$	
Tolerance for solver	$CTOL := TOL$	
Start solver block	Given	
equation of circle	$x^2 + y^2 = r^2$	
equation of flared portion	$y = \frac{L_{tower}}{2} + \tan(\theta_{spread}) \cdot \left(x - \frac{L_{tower}}{2}\right)$	
Solution to equations	$soln := Find(x, y)$	
x coordinate of intersection	$x_{intersect} := soln_0$	$x_{intersect} = 511.35\,ft$
y coordinate of intersection	$y_{intersect} := soln_1$	$y_{intersect} = 213.15\,ft$
chord length	$L_{chord} := 2 \cdot y_{intersect}$	$L_{chord} = 426.30\,ft$
distance from tower face to petal	$h := x_{intersect} - \frac{L_{tower}}{2}$	$h = 407.35\,ft$
Subtending angle of petal	$\theta := 2\,asin\left(\frac{L_{chord}}{2 \cdot r}\right)$	$\theta = 45.26\,deg$
Area of oustide of chord	$A_{outside} := \frac{r^2}{2} \cdot (\theta - \sin(\theta))$	$A_{outside} = 1.22 \times 10^4\,ft^2$
Area inside of chord	$A_{inside} := \frac{h}{2} \cdot (L_{tower} + L_{chord})$	$A_{inside} = 1.29 \times 10^5\,ft^2$
Area of one petal	$A_{1.petal} := A_{inside} + A_{outside}$	$A_{1.petal} = 1.41 \times 10^5\,ft^2$

Calculate area in corner

Reuse analysis for Maltese cross petals, but with radius set to inner circle radius

Solve for intersection of circle and line representing flare
Origin is at center of circle

Radius of interest	$r := r_{inner}$	$r = 400.00\ ft$
Initial guess for x	$x := r$	
Initial guess for y	$y := 0ft$	
Tolerance for solver	$TOL := 1 \cdot 10^{-6}$	
Tolerance for solver	$CTOL := TOL$	
Start solver block	Given	
equation of circle	$x^2 + y^2 = r^2$	
equation of flared portion	$y = \frac{L_{tower}}{2} + \tan(\theta_{spread}) \cdot \left(x - \frac{L_{tower}}{2}\right)$	
Solution to equations	$soln := Find(x, y)$	
x coordinate of intersection	$x_{intersect} := soln_0$	$x_{intersect} = 360.75\ ft$
y coordinate of intersection	$y_{intersect} := soln_1$	$y_{intersect} = 172.80\ ft$
chord length	$L_{chord} := 2 \cdot y_{intersect}$	$L_{chord} = 345.59\ ft$
distance from tower face to petal	$h := x_{intersect} - \frac{L_{tower}}{2}$	$h = 256.75\ ft$
Subtending angle of petal	$\theta := 2\,asin\left(\frac{L_{chord}}{2 \cdot r}\right)$	$\theta = 51.19\,deg$
Area of oustide of chord	$A_{outside} := \frac{r^2}{2} \cdot (\theta - \sin(\theta))$	$A_{outside} = 9.14 \times 10^3\ ft^2$
Area inside of chord	$A_{inside} := \frac{h}{2} \cdot (L_{tower} + L_{chord})$	$A_{inside} = 7.11 \times 10^4\ ft^2$
Area of one petal	$A_{1.petal.for.inner} := A_{inside} + A_{outside}$	$A_{1.petal.for.inner} = 8.02 \times 10^4\ ft^2$
Area of inner circle	$A_{inner.circle} := \pi \cdot r_{inner}^2$	$A_{inner.circle} = 5.03 \times 10^5\ ft^2$
Area of tower	$A_{tower} := L_{tower}^2$	$A_{tower} = 4.33 \times 10^4\ ft^2$
Area of 4 corners	$A_{4.corners} := A_{inner.circle} - 4A_{1.petal.for.inner} - A_{tower}$	$A_{4.corners} = 1.39 \times 10^5\ ft^2$
Area of 1 corner	$A_{1.corner} := \frac{A_{4.corners}}{4}$	$A_{1.corner} = 3.46 \times 10^4\ ft^2$

Calculate number of trees

Number of trees that "peeled"	$N_{peel} := \frac{h_{peel}}{h_{total}} \cdot N_{tree.per.face}$	$N_{peel} = 236.59$
Remainder of trees	$N_{remainder} := N_{tree.per.face} - N_{peel}$	$N_{remainder} = 463.41$
Number of trees in that are in the overlap area of the petal and inner circle	$N_{overlap} := Ratio_{overlap.to.others} \cdot N_{remainder}$	$N_{overlap} = 323.46$
Number of trees in the corner	$N_{corner} := N_{remainder} - N_{overlap}$	$N_{corner} = 139.95$

Calculate tree density

Tree density that "peeled"	$\rho_{peel} := \frac{N_{peel}}{A_{1.petal}}$	$\rho_{peel} = 1.67 \times 10^{-3}\ ft^{-2}$
Tree density in the overlap area	$\rho_{overlap} := \frac{N_{overlap}}{A_{1.petal.for.inner}}$	$\rho_{overlap} = 4.03 \times 10^{-3}\ ft^{-2}$
Tree density in the corner	$\rho_{corner} := \frac{N_{corner}}{A_{1.corner}}$	$\rho_{corner} = 4.04 \times 10^{-3}\ ft^{-2}$

Pixel Counts

WTC footprint	$Pixels_{WTC.Footprint} := 187893$	
WTC 1, peeled area	$Pixels_{WTC.1.peeled} := 79190$	
WTC 1, corner area	$Pixels_{WTC.1.Corner} := 50724$	
WTC 1, overlap area	$Pixels_{WTC.1.Overlap} := 25308$	
WTC 2, peeled area	$Pixels_{WTC.2.peeled} := 79774$	
WTC 2, corner area	$Pixels_{WTC.2.Corner} := 52693$	
WTC 2, overlap area	$Pixels_{WTC.2.Overlap} := 25948$	
Ratio of squre feet to pixels	$Ratio_{sq.ft.to.pixels} := \frac{L_{tower}^{2}}{Pixels_{WTC.Footprint}}$	$Ratio_{sq.ft.to.pixels} = 0.23\ ft^{2}$

Areas

WTC 1, peeled area	$A_{WTC.1.peeled} := Ratio_{sq\ ft.to.pixels} \cdot Pixels_{WTC.1.peeled}$	$A_{WTC.1.peeled} = 1.82 \times 10^4\ ft^2$
WTC 1, corner area	$A_{WTC.1.Corner} := Ratio_{sq.ft\ to.pixels} \cdot Pixels_{WTC.1\ Corner}$	$A_{WTC.1.Corner} = 1.17 \times 10^4\ ft^2$
WTC 1, overlap area	$A_{WTC.1.Overlap} := Ratio_{sq.ft.to.pixels} \cdot Pixels_{WTC\ 1.Overlap}$	$A_{WTC.1.Overlap} = 5.83 \times 10^3\ ft^2$
WTC 2, peeled area	$A_{WTC.2.peeled} := Ratio_{sq.ft\ to.pixels}\ Pixels_{WTC\ 2.peeled}$	$A_{WTC.2\ peeled} = 1.84 \times 10^4\ ft^2$
WTC 2, corner area	$A_{WTC.2.Corner} := Ratio_{sq.ft.to.pixels} \cdot Pixels_{WTC.2\ Corner}$	$A_{WTC.2.Corner} = 1.21 \times 10^4\ ft^2$
WTC 2, overlap area	$A_{WTC.2.Overlap} := Ratio_{sq.ft.to.pixels} \cdot Pixels_{WTC.2\ Overlap}$	$A_{WTC.2.Overlap} = 5.97 \times 10^3\ ft^2$

Number of trees impacted

WTC 1, peeled area	$N_{WTC.1.peeled} := A_{WTC.1.peeled} \cdot \rho_{peel}$	$N_{WTC.1.peeled} = 30.5$
WTC 1, corner area	$N_{WTC.1.Corner} := A_{WTC.1.Corner} \cdot \rho_{corner}$	$N_{WTC.1.Corner} = 47.2$
WTC 1, overlap area	$N_{WTC.1.Overlap} := A_{WTC.1.Overlap} \cdot \rho_{overlap}$	$N_{WTC.1.Overlap} = 23.5$
WTC 1, total	$N_{WTC.1.Total} := N_{WTC.1.peeled} + N_{WTC\ 1.Corner} + N_{WTC.1.Overlap}$	
		$N_{WTC.1.Total} = 101.2$
WTC 2, peeled area	$N_{WTC.2.peeled} := A_{WTC.2.peeled} \cdot \rho_{peel}$	$N_{WTC.2\ peeled} = 30.7$
WTC 2, corner area	$N_{WTC.2\ Corner} := A_{WTC.2.Corner} \cdot \rho_{corner}$	$N_{WTC.2.Corner} = 49.0$
WTC 2, overlap area	$N_{WTC.2.Overlap} := A_{WTC.2.Overlap}\ \rho_{overlap}$	$N_{WTC.2.Overlap} = 24.1$
WTC 2, total	$N_{WTC.2.Total} := N_{WTC.2.peeled} + N_{WTC\ 2\ Corner} + N_{WTC.2.Overlap}$	
		$N_{WTC.2.Total} = 103.8$

Summary

Percentage of debris not counted in "peeled" area that fell into the "overlapped" area	$Ratio_{overlap.to.others} = 69.80\%$
Number of trees impacted, WTC 1	$N_{WTC.1.Total} = 101.2$
Number of trees impacted, WTC 2	$N_{WTC.2.Total} = 103.8$